



A13.2 Function types and function application

Functional programming · A level · AQA 7517 4.12.1.1 · about 20 min

What this lesson is about

Functions as mappings, function type $f: A \rightarrow B$, domain and co-domain, and function application with arguments from a Cartesian product.

- A function is a mapping
- Function type, domain and co-domain
- Writing the type in code
- Function application
- Applying a type to the robot

Questions

6 marks in all

1. A function has type $f: A \rightarrow B$. What is B called? [1 mark]

- ☐ A The co-domain
- ☐ B The domain
- ☐ C The argument type
- ☐ D The Cartesian product

Answer: A. A is the argument type, called the domain; B is the result type, called the co-domain.

2. `is_even` takes a whole number and returns True or False. What is its function type? [1 mark]

- ☐ A `is_even: integer → Boolean`
- ☐ B `is_even: Boolean → integer`
- ☐ C `is_even: integer × Boolean → integer`
- ☐ D `is_even: integer → integer`

Answer: A. The argument type comes before the arrow and the result type after it.

3. Which statements about a function $f: A \rightarrow B$ are true? [1 mark]

Tick every answer that is true.

- ☐ A A and B are subsets of objects in some data type
- ☐ B Each value in A is mapped to exactly one value in B
- ☐ C Every value in B must be the result for some value in A
- ☐ D A is the set of results the function gives

Answer: A, B. The co-domain is the set results are taken from; a function need not produce every value in it. The domain, not the co-domain, is the set of arguments.

4. `add` has type `integer × integer → integer`. What is `integer × integer` called? [1 mark]

Answer: Cartesian product. `integer × integer` is the Cartesian product of the integers with itself: the set of all pairs of integers.

5. In function application, how many arguments does `add(3, 4)` strictly take, if `add: integer × integer → integer`? [1 mark]
- ☐ A One, the pair (3, 4)
 - ☐ B Two, 3 and 4
 - ☐ C None, it is a constant
 - ☐ D Three, 3, 4 and the result

Answer: A. The domain is the Cartesian product $\text{integer} \times \text{integer}$, so its single argument is a pair.

6. What does this program print? [1 mark]

```
def compass(degrees):  
    return "NESW"[((degrees + 45) % 360) // 90]  
  
print(compass(44), compass(45), compass(224), compass(315))
```

Answer:

N E S N

Adding 45 and dividing by 90 gives the quarter: 44 is still N, 45 becomes E, 224 is S, and 315 wraps round to N.

The task: the compass type

Write `compass` with its type hints, exactly as `def compass(degrees: int) -> str: .` Its domain is the whole numbers 0 to 359, and it returns "N" for 0 to 44 and 315 to 359, "E" for 45 to 134, "S" for 135 to 224 and "W" for 225 to 314. Then: 1. Apply `compass` to **every** value in its domain, collect the different results, and print them sorted alphabetically and separated by spaces, as `range: <letters> .` Work the letters out; do not type them into the `print` . 2. Turn the robot right by 90 degrees four times. After each turn, print `facing <letter>` , where the letter is `compass` applied to the robot's `heading()` rounded to a whole number and taken MOD 360.

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()

def compass(degrees):
    return "N"

print("range:", compass(0))
```

The hint students can ask for: Work out a way to turn any whole number of degrees into a quarter number 0 to 3, so the four ranges line up with N, E, S and W; check the edges 44, 45, 314 and 315. For the range, a set keeps only the different results of applying the function to every value in the domain.

A solution

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()

def compass(degrees: int) -> str:
    return "NESW"[((degrees + 45) % 360) // 90]

results = set(map(compass, range(360)))
print("range:", " ".join(sorted(results)))

for i in range(4):
    turn_right(30, angle=90)
    print("facing", compass(round(heading()) % 360))
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.