



A3.7 Vectors

Data structures · A level · AQA 7517 4.2.1.4 · about 25 min

Name _____

Class _____

Date _____

What this lesson is about

Vectors as lists, functions and arrows; addition, scaling, dot product and convex combination, on the robot's position.

- Three ways to see a vector
- Adding and scaling
- Convex combination
- Dot product
- BugBot's vectors

Questions

6 marks in all

1. What is the dot product of $[2, -1, 3]$ and $[4, 5, 1]$? [1 mark]

2. What does this program print? [1 mark]

```
u = [2, 5]
v = [6, 1]
w = [0.5 * u[i] + 0.5 * v[i] for i in range(2)]
print(w)
```

3. For $\alpha u + \beta v$ to be a convex combination of u and v , which conditions must hold? [1 mark]

- ☐ A $\alpha \geq 0, \beta \geq 0$ and $\alpha + \beta = 1$
- ☐ B $\alpha + \beta = 0$
- ☐ C $\alpha = \beta$
- ☐ D $\alpha > 1$ and $\beta > 1$

4. The dot product of two non-zero vectors is 0. What does that tell you? [1 mark]

- ☐ A They are perpendicular
- ☐ B They point in the same direction
- ☐ C They have the same magnitude
- ☐ D One of them is the zero vector

5. The vector $[2.0, 3.5, -1.0]$ is a member of which set? [1 mark]

- ☐ A $\mathbb{R}^3$
- ☐ B $\mathbb{R}^2$
- ☐ C $\mathbb{N}^3$
- ☐ D $\mathbb{R}^1$

6. What is the angle, in degrees, between the vectors $[1, 0]$ and $[1, 1]$? [1 mark]

The task: meet on the line

Two beacons are at $u = [40, 20]$ and $v = [-20, 60]$, in cm from the robot's start (x to the right, y forward). Write these three functions yourself, for vectors of any length, without numpy: - `add(a, b)`: returns a new list, the vector sum of a and b . - `scale(k, a)`: returns a new list, the number k times the vector a . - `dot(a, b)`: returns the dot product of a and b , a single number. Then print, in this order: 1. $u \cdot v = \langle \text{dot product} \rangle$, which is a whole number. 2. $\text{angle} = \langle \text{angle} \rangle$ degrees, the angle between u and v rounded to a whole number with `round()`. You will need `math.acos`, `math.degrees` and `math.sqrt`. 3. $w = \langle w \rangle$, where $w = 0.25u + 0.75v$, built with `scale` and `add`, printed as a Python list, for example $w = [1.0, 2.0]$. Finally drive to w : slide sideways by its x component (left if it is negative) and then forward by its y component.

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()
import math

u = [40, 20]
v = [-20, 60]
```

Plan your program here, then type it in and press Run.



Do it on the robot

www.bugbotlab.com/learn/a3-7-vectors/

The simulator checks it and tells you when it passes. Nothing to install, no account.

Challenges

1. Is w really on the segment between the beacons? Check that $w - u$ is a positive scalar multiple of $v - u$.
2. Find two different whole-number vectors whose dot product with $[3, 4]$ is zero. What do they have in common?
3. Rewrite `dot` so it takes two vectors stored as dictionaries, where missing indexes mean zero.