



A6.3 Sets

What this lesson is about

Set notation and comprehension, finite and countably infinite sets, cardinality, Cartesian product, subsets, union, intersection and difference.

- What a set is
- Set comprehension
- Finite and infinite sets, and cardinality
- Cartesian product
- Subsets and set operations

Questions

6 marks in all

1. $A = \{3, 5, 7\}$ and $B = \{a, b, c, d\}$. What is $|A \times B|$? [1 mark]

Answer: 12. The Cartesian product has one pair for every member of A with every member of B : $3 \times 4 = 12$.

2. $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$. List $A \setminus B$ (the difference), as numbers separated by commas in ascending order. [1 mark]

Answer: 1, 2. $A \setminus B$ is the members of A that are not in B .

3. Which set is $\{2x \mid x \in \mathbb{N} \wedge x < 4\}$? [1 mark]

- ☐ A $\{0, 2, 4, 6\}$
☐ B $\{2, 4, 6, 8\}$
☐ C $\{0, 1, 2, 3\}$
☐ D $\{2, 4, 6\}$

Answer: A. x takes the natural numbers 0, 1, 2 and 3, and each is doubled.

4. Which of these sets are countably infinite? [1 mark]

Tick every answer that is true.

- ☐ A The natural numbers $\mathbb{N}$
☐ B The integers $\mathbb{Z}$
☐ C $\{x \mid x \in \mathbb{N} \wedge x < 1000\}$
☐ D The set of all binary strings

Answer: A, B, D. $\mathbb{N}$, $\mathbb{Z}$ and all binary strings can each be counted off 0, 1, 2, ... The third set is finite, with 1000 members.

5. $A = \{1, 3, 5\}$. Which statement is false?

[1 mark]

- ☐ A $\{1, 5\} \subset A$
- ☐ B $A \subseteq A$
- ☐ C $A \subset A$
- ☐ D $\{\} \subseteq A$

Answer: C. A proper subset must be missing at least one member of the other set, so no set is a proper subset of itself.

6. What does this program print?

[1 mark]

```
A = {1, 3, 7, 9}
B = {2, 3, 9}
print(sorted(A | B))
print(sorted(A & B))
print(len(A - B), len(B - A))
```

Answer:

```
[1, 2, 3, 7, 9]
[3, 9]
2 1
```

Union has every member of either set; intersection only those in both. $A - B = \{1, 7\}$ and $B - A = \{2\}$.

The task: two robots' sightings

Two robots patrolled the same room and logged the marker ids they saw, with repeats. Let A be the set of ids robot A saw and B the set robot B saw. - Start from the two lists in the starter. Do not type any of the answers. - Print exactly these eight lines, in this order. A list of ids is written in ascending order separated by single spaces: 1. A: then the members of A 2. B: then the members of B 3. union: then the members of $A \cup B$ 4. intersection: then the members of $A \cap B$ 5. A minus B: then the members of $A \setminus B$ 6. pairs: then the cardinality of $A \times B$, as a whole number 7. subset: then True or False: whether $\{7, 9\}$ is a proper subset of A 8. even: then the members of $\{x \mid x \in A \cup B \wedge x \text{ is even}\}$, built with a set comprehension For example, if A were $\{2, 5\}$ the first line would be A: 2 5.

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()

seen_by_a = [3, 7, 1, 7, 9, 3]
seen_by_b = [7, 2, 9, 9, 4]
```

The hint students can ask for: Turn each list into a set first, so the repeats disappear. Each line is then one set operation or one comparison. To print a set in the required form, put it in order and join the numbers as text. The number of pairs in a Cartesian product is the product of the two cardinalities.

A solution

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()

seen_by_a = [3, 7, 1, 7, 9, 3]
seen_by_b = [7, 2, 9, 9, 4]

def show(s):
    return " ".join(str(x) for x in sorted(s))

A = set(seen_by_a)
B = set(seen_by_b)
print("A:", show(A))
print("B:", show(B))
print("union:", show(A | B))
print("intersection:", show(A & B))
print("A minus B:", show(A - B))
print("pairs:", len(A) * len(B))
print("subset:", {7, 9} < A)
print("even:", show({x for x in A | B if x % 2 == 0}))
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.