



A8.9 Project: the robot's safety logic

Boolean algebra and logic circuits · A level · OCR H446 1.4.3, AQA 7517

4.6.4.1, Eduqas A500QS 1.2 · about 30 min

What this lesson is about

From a rule table to a Karnaugh map, a simplified expression, a proof and a robot that stops for the right reasons.

- The brief
- Stage 1: the truth table
- Stage 2: the sum of products
- Stage 3: the Karnaugh map
- Stage 4: simplify further, and prove it
- Stage 5: wire it to the robot

Questions

5 marks in all

1. A rule has four inputs. How many rows does its truth table have? [1 mark]

Answer: 16. 2 to the power 4 is 16.

2. With rows numbered $8C + 4K + 2L + D$, which row number is $C = 1, K = 0, L = 1, D = 1$? [1 mark]

Answer: 11. $8 + 0 + 2 + 1 = 11$, the binary number 1011.

3. Put the stages of designing a logic rule in order. [1 mark]

Number the lines 1 to 6 to put them in the right order.

- ___ Write the simplified expression
- ___ Draw the circuit or write the code
- ___ Write the truth table
- ___ Name each input with a letter
- ___ Check it against the truth table
- ___ Draw the Karnaugh map and group the 1s

Answer:

Name each input with a letter
 Write the truth table
 Draw the Karnaugh map and group the 1s
 Write the simplified expression
 Check it against the truth table
 Draw the circuit or write the code

From the problem to a table, from the table to a simple expression, then prove it before building it.

4. Two groups on a map give $(\neg K \wedge D) \vee (\neg K \wedge \neg C \wedge \neg L)$. Which law turns this into $\neg K \wedge (D \vee (\neg C \wedge \neg L))$? [1 mark]
- ☐ A Distribution
 - ☐ B De Morgan's law
 - ☐ C Double negation
 - ☐ D Commutation

Answer: A. Distribution in reverse takes the shared $\neg K$ out of both terms.

5. Which expression is equal to $\neg C \wedge \neg L$? [1 mark]
- ☐ A $\neg(C \vee L)$
 - ☐ B $\neg(C \wedge L)$
 - ☐ C $C \vee L$
 - ☐ D $\neg C \vee \neg L$

Answer: A. De Morgan: $\neg(C \vee L) = \neg C \wedge \neg L$, which saves one NOT gate.

The task: the robot's safety logic

The starter gives `GO_ROWS`, the truth table rows (numbered $8C + 4K + 2L + D$) where $G = 1$, and `draft(c, k, l, d)`, which takes four bits and returns 1 on those rows and 0 otherwise. 1. Write `go(c, k, l, d)`, taking four bits, as a single `return` line using at most six of `and`, `or` and `not` in total. It may not use `GO_ROWS` or call `draft`. Any true result means drive. 2. Check `go` against `draft` on all sixteen combinations, treating a true result as 1 and a false one as 0, and print `matches: True` if they agree on every one, or `matches: False` if not. 3. Drive the robot. Set `docking = 0`. In a loop, read `close` (1 if `distance()` is under 20, else 0), `hit` (1 if `bumped()`, else 0) and `low` (1 if `battery()` is under 20, else 0), and call `go(close, hit, low, docking)`. While it is true, drive forward. When it is false, stop, print one line in exactly the form `stopped: C=1 K=0 L=0 D=0` using the values that stopped it, and end the loop. The robot must finish in the band before the wall without touching it.

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()

GO_ROWS = [0, 1, 3, 9, 11]

def draft(c, k, l, d):
    return 1 if 8 * c + 4 * k + 2 * l + d in GO_ROWS else 0

def go(c, k, l, d):
    return 0
```

The hint students can ask for: Put the five rows on a four-variable Karnaugh map in Gray code order and find the fewest, largest groups; then see whether factoring out a shared literal, or De Morgan, saves operators. Prove `go` against `draft` on all sixteen rows before you let it drive. In the loop, read the four inputs as 0 or 1 every time round.

A solution

```
# the two lines every program starts with: the commands, then the robot
from bugbot import *
connect()

GO_ROWS = [0, 1, 3, 9, 11]

def draft(c, k, l, d):
    return 1 if 8 * c + 4 * k + 2 * l + d in GO_ROWS else 0

def go(c, k, l, d):
    return (not k) and (d or not (c or l))

ok = True
for n in range(16):
    c, k, l, d = n >> 3 & 1, n >> 2 & 1, n >> 1 & 1, n & 1
    if (1 if go(c, k, l, d) else 0) != draft(c, k, l, d):
        ok = False
print(f"matches: {ok}")

docking = 0
while True:
    close = 1 if distance() < 20 else 0
    hit = 1 if bumped() else 0
    low = 1 if battery() < 20 else 0
    if go(close, hit, low, docking):
```

```
        forward(50)
        wait(0.1)
    else:
        stop()
        print(f"stopped: C={close} K={hit} L={low} D={docking}")
        break
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.