



U11.1 The pinhole camera

What this lesson is about

One pixel is one ray. What that buys you, and what it does not.

- The model
- What a pixel is worth
- The tangent matters
- A bearing is not a position
- Getting the depth back

Questions

7 marks in all

1. The BugBot's camera is 320 pixels wide with a 120 degree field of view. What does this print? [1 mark]

```
import math
f = (320 / 2) / math.tan(math.radians(120 / 2))
bearing = math.degrees(math.atan((240 - 160) / f))
print(round(f, 1), round(bearing, 1))
```

Answer:

92.4 40.9

$f = 160 / \tan(60 \text{ degrees}) = 92.4 \text{ px}$, and column 240 is $\text{atan}(80 / 92.4) = 40.9 \text{ degrees}$ right of the nose.

2. Which line works out the focal length in pixels correctly? [1 mark]

- ☐ A $f = 160 / \text{math.tan}(\text{math.radians}(60))$
- ☐ B $f = 160 / \text{math.tan}(60)$
- ☐ C $f = 160 * \text{math.tan}(\text{math.radians}(60))$
- ☐ D $f = 320 / \text{math.tan}(\text{math.radians}(120))$

Answer: A. math.tan takes radians, so 60 must be converted first. Half the width goes with half the field of view, and the focal length divides by the tangent.

3. A blob's centre is at column 200. What is its bearing from straight ahead, in degrees to 1 decimal place, using $f = 92.4$? [1 mark]

Answer: 23.4 (accept within 0.1). $\text{bearing} = \text{atan}((200 - 160) / 92.4) = \text{atan}(0.433) = 23.4 \text{ degrees}$.

4. The linear guess bearing = $(u - 160) \times 0.375$ degrees is used instead of the pinhole formula. How many degrees wrong is it at column 200? Give 1 decimal place. [1 mark]

Answer: 8.4 (accept within 0.1). The linear guess gives $40 \times 0.375 = 15.0$ degrees, the pinhole gives 23.4, so it is 8.4 degrees short. They only agree at the centre and the very edge.

5. Why can a single pixel not tell you where an object is? [1 mark]
- ☐ A Depth appears only as a divisor, so every point along a ray lands on the same pixel
 - ☐ B The image is too low resolution to locate the object
 - ☐ C The principal point is not exactly at the centre
 - ☐ D Lens distortion moves the pixel

Answer: A. Doubling X and Z gives the same u. A pixel names a direction, and the range has to come from a second constraint.

6. Which of these add the second constraint needed to get depth from a camera? [1 mark]

Tick every answer that is true.

- ☐ A Knowing the object's real size
- ☐ B Knowing the object is on a flat floor, with the camera's height and tilt
- ☐ C A second camera a known distance away
- ☐ D The same camera at two times, with the motion between them known
- ☐ E Averaging the bearing over many frames
- ☐ F A higher resolution sensor

Answer: A, B, C, D. All four add a second ray or a known size. Averaging and more pixels make the bearing better but still give only a direction.

7. A tag is seen at column 220 and the detector reports it 50 cm away. What does this print? [1 mark]

```
import math
F = 92.4
bearing = math.degrees(math.atan((220 - 160) / F))
print(round(bearing, 1), round(50 * math.sin(math.radians(bearing)), 1), round(50 *
math.cos(math.radians(bearing)), 1))
```

Answer:

33.0 27.2 41.9

The bearing is $\text{atan}(60 / 92.4) = 33.0$ degrees, so the tag is $50 \sin 33.0 = 27.2$ cm right and $50 \cos 33.0 = 41.9$ cm ahead.

The task: turn a pixel into a bearing

Tag 7 is on the mat and the robot is standing still, facing along +y. Print `bearing:` , the angle from straight ahead to the tag in degrees, and `across:` , how far to the right of the robot's nose line the tag actually is in centimetres.

```
from bugbot import *
import math
connect()

F = 92.4
set_cv("apriltag")
wait(0.3)
```

The hint students can ask for: The detector gives you a column in the picture and a range. The column is an angle: the camera is 320 pixels wide across 120 degrees, and the relation between a pixel offset from the centre and an angle is a tangent, not a straight line. The sideways offset needs the range as well, because a bearing on its own is a whole ray of possible places.

A solution

```
from bugbot import *
import math
connect()

F = 92.4                                # focal length in pixels: 160 / tan(60 degrees)

set_cv("apriltag")
wait(0.3)
tags = [t for t in apriltags() if t[0] == 7]
tag_id, cx, cy, dist = tags[0]

bearing = math.degrees(math.atan((cx - 160) / F))
across = dist * math.sin(math.radians(bearing))
print("bearing:", round(bearing, 2))
print("across:", round(across, 1))
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.