ANSWERS

U4.1 A reading is a distribution

Noise and filtering · University · about 25 min

What this lesson is about

Mean, standard deviation, and what one number from a sensor is actually telling you.

- Two numbers describe it
- Measuring it
- Bias against noise, again
- A new reading only arrives so often

Questions

6 marks in all

1. A still robot's depth readings have mean 57.0 cm and standard deviation 2.0 cm, and are Gaussian. About what fraction of readings land between 55 and 59 cm? [1 mark]

- ☐ **A** About 68 percent
- ☐ **B** About 95 percent
- ☐ **C** About 50 percent
- ☐ **D** About 99.7 percent

> **Answer:** A. 55 to 59 cm is the mean plus or minus one sigma, which holds about 68 percent of a Gaussian. Plus or minus two sigma (53 to 61 cm) would be 95 percent.

2. Same sensor: mean 57.0 cm, sigma 2.0 cm. About what percentage of readings fall outside 51 to 63 cm? Give a percentage to one decimal place. [1 mark]

> **Answer:** 0.3 (accept within 0.05). 51 to 63 cm is three sigma either side, which holds 99.7 percent, so about 0.3 percent fall outside. That is what engineers mean by three sigma: practically never.

3. This computes the mean and sigma exactly as the lesson does. What does it print? [1 mark]

```
readings = [56, 58, 60, 62, 64]
mean = sum(readings) / len(readings)
var = sum((r - mean) ** 2 for r in readings) / len(readings)
sigma = var ** 0.5
print(mean, var, round(sigma, 2))
```

> **Answer:**
>
> ```
> 60.0 8.0 2.83
> ```
>
> The mean is 60. The squared deviations are 16, 4, 0, 4 and 16, averaging 8, which is the variance in cm squared. Sigma is its square root, 2.83 cm.

4. A sensor gives readings with sigma 0.3 cm, but their mean is 4 cm short of a tape-measured distance. What will fix it? [1 mark]

- ☐ **A** Calibrate against the tape measure
- ☐ **B** Average more readings
- ☐ **C** Use a stronger low pass filter
- ☐ **D** Take the readings further apart in time

> **Answer:** A. Small sigma and a shifted mean is bias, not noise. Averaging and filtering only reduce scatter, so only calibration against something you trust removes the shift.

5. A student reads the depth sensor 100 times in a tight loop with no wait and gets sigma = 0.4 cm, much smaller than with a 0.1 s wait. Why is that number a lie? [1 mark]

- ☐ **A** The sensor only produces a new measurement about every 0.1 s, so most reads repeat the same sample
- ☐ **B** Reading quickly lets the sensor warm up and become more precise
- ☐ **C** The variance should have been divided by 99, not 100
- ☐ **D** Sigma is the square of the variance, so it was computed wrongly

> **Answer:** A. A hundred reads between updates is a handful of samples counted many times, so the spread looks small. Independence comes from time between readings.

6. A reading's spread is described by its standard deviation. What is the name of its square, measured in squared units? [1 mark]

> **Answer:** variance. Variance is the mean squared deviation, in cm squared. Sigma is its square root, in cm, which is why sigma is the one that compares directly with a reading.

The task: measure the noise

Standing still, print `mean:` and `sigma:` for this sensor, in centimetres.

```
from bugbot import *
connect()

readings = []
```

The hint students can ask for: Stand still and read distance() a hundred times, a tenth of a second apart. The mean is the sum over the count; sigma is the square root of the mean squared difference from that mean.

A solution

```
from bugbot import *
connect()

readings = []
for i in range(100):
    readings.append(distance())
    wait(0.1)

mean = sum(readings) / len(readings)
var = sum((r - mean) ** 2 for r in readings) / len(readings)
print("mean:", round(mean, 1))
print("sigma:", round(var ** 0.5, 2))
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.