ANSWERS

U4.5 The price of filtering

Noise and filtering · University · about 30 min

What this lesson is about

Every filter delays. Measuring the delay, and what it does to a loop closed around it.

- How much delay
- Measure it
- Why it matters so much in a loop
- What to do instead

Questions

6 marks in all

1. An exponential filter with alpha = 0.2 runs in a 0.1 s loop. About how many seconds does it delay the signal? [1 mark]

> **Answer:** 0.4 (accept within 0.01). The delay is (1 - alpha) / alpha = 0.8 / 0.2 = 4 steps, and 4 x 0.1 s = 0.4 s.

2. What does this print? [1 mark]

```
DT = 0.1
for a in (0.5, 0.25, 0.1):
    steps = (1 - a) / a
    print(a, round(steps, 2), round(steps * DT, 2))
```

> **Answer:**
>
> ```
> 0.5 1.0 0.1
> 0.25 3.0 0.3
> 0.1 9.0 0.9
> ```
>
> (1 - alpha) / alpha gives 1, 3 and 9 steps, so 0.1 s, 0.3 s and 0.9 s at a 0.1 s loop. Halving alpha more than doubles the delay.

3. A controller holds a gap to the wall. Why should the filter go on the measurement rather than on the error? [1 mark]

- ☐ **A** Filtering the error also delays the response to a change in the target
- ☐ **B** Filtering the error removes more noise
- ☐ **C** The error has no noise in it
- ☐ **D** Filtering the measurement has no delay

> **Answer:** A. The error contains the setpoint as well as the measurement, so filtering it slows the reaction to a new target. Only the measurement is noisy.

4. In a PID controller, which term usually needs the filter? [1 mark]

- ☐ **A** The D term
- ☐ **B** The P term
- ☐ **C** The I term
- ☐ **D** All three equally

> **Answer:** A. Differentiating amplifies noise, so D is where filtering pays. The I term already averages, and P usually tolerates the raw noise.

5. The plant has a time constant of 0.25 s and the loop runs at 0.1 s. By the lesson's rule of thumb, which filter is acceptable in the loop? [1 mark]

- ☐ **A** alpha = 0.5, a delay of 0.1 s
- ☐ **B** alpha = 0.1, a delay of 0.9 s
- ☐ **C** alpha = 0.05, a delay of 1.9 s
- ☐ **D** Any alpha, since delay does not affect stability

> **Answer:** A. The filter delay must be small compared with tau. 0.1 s against 0.25 s is acceptable; 0.9 s and 1.9 s are longer than the thing being controlled.

6. A controller was twitchy, so its sensor filter was made heavier twice. It now oscillates slowly. What is the right fix? [1 mark]

- ☐ **A** Filter less, because the added delay is now causing the oscillation
- ☐ **B** Filter more, to smooth out the oscillation
- ☐ **C** Raise the gain so it responds faster
- ☐ **D** Filter the error as well as the measurement

> **Answer:** A. Delay in the feedback path turns negative feedback into positive feedback. Same symptom as before, opposite fix: take delay out.

The task: measure the lag

Drive steadily at the wall, filter the distance, plot `raw` and `filtered`, and print `lag:`, the seconds by which the filtered signal trails the raw one.

```
from bugbot import *
connect()

ALPHA = 0.2
DT = 0.1
filtered = distance()
```

The hint students can ask for: Drive steadily at the wall so the distance falls in a straight line, then compare the two signals: the filtered one reaches any given value later. The theory says the delay is about (1 - alpha) over alpha, times the loop period. Check it against what you measure.

A solution

```
from bugbot import *
connect()

ALPHA = 0.2
DT = 0.1
filtered = distance()
raws, filts = [], []
forward(50)
for i in range(60):
    raw = distance()
    filtered = ALPHA * raw + (1 - ALPHA) * filtered
    raws.append(raw)
    filts.append(filtered)
    plot("raw", raw)
    plot("filtered", filtered)
    if raw < 30:
        break
    wait(DT)
stop()

# how much later does the filtered signal reach the value the raw one reached?
target = raws[len(raws) // 2]
def first_below(series):
    for i, v in enumerate(series):
        if v <= target:
            return i
    return len(series)

lag = (first_below(filts) - first_below(raws)) * DT
print("theory:", round((1 - ALPHA) / ALPHA * DT, 2))
print("lag:", round(max(lag, 0.0), 2))
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.