



U6.5 Covariance and the ellipse

What this lesson is about

Uncertainty in two dimensions has a shape, and the shape is the useful part.

- Why it is not a circle
- The covariance matrix
- Why this earns its keep
- In practice

Questions

6 marks in all

1. A robot drives 200 cm straight with a 1 degree heading error. How large, in cm to one decimal place, is the across-track error? [1 mark]

Answer: 3.5 (accept within 0.06). Across-track error is heading error in radians times distance: $200 \times 0.01745 = 3.49$ cm.

2. In two dimensions, about what percentage of the probability lies inside the one sigma ellipse? [1 mark]

Answer: 39 (accept within 1). About 39 percent, not the 68 percent of one dimension. The 95 percent ellipse is about 2.45 sigma.

3. The ellipse axes of a covariance matrix $\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$, from the 2×2 eigenvalue formula. What does it print? [1 mark]

```
a, b, d = 5.0, 2.0, 2.0
tr, det = a + d, a * d - b * b
root = (tr * tr / 4 - det) ** 0.5
l1, l2 = tr / 2 + root, tr / 2 - root
print(l1, l2)
print(round(l1 ** 0.5, 2), round(l2 ** 0.5, 2))
```

Answer:

```
6.0 1.0
2.45 1.0
```

The trace is 7 and the determinant 6, so the eigenvalues are 6 and 1. The one sigma axes are their square roots, 2.45 and 1: a tilted ellipse, because the off-diagonal is not zero.

4. A tag straight ahead of the robot gives a good range and a poor bearing. Which part of the uncertainty ellipse does a fix mainly shrink? [1 mark]
- ☐ A Along the line of sight
- ☐ B Across the line of sight
- ☐ C Both equally
- ☐ D Neither, bearings are needed to shrink anything

Answer: A. A range measures distance along the line to the tag. A filter carrying the full matrix takes exactly that direction from the fix and leaves the sideways uncertainty.

5. What does the off-diagonal term $\text{cov}(x, y)$ of P tell you?

[1 mark]

- ☐ A Whether the x and y errors move together, which tilts the ellipse
- ☐ B How large the x uncertainty is
- ☐ C The total uncertainty
- ☐ D The heading error

Answer: A. The diagonal gives each axis's variance. The off-diagonal says whether the errors are correlated, which rotates the ellipse away from the axes.

6. The uncertainty ellipse has grown long across the direction of travel. What kind of landmark would help most? [1 mark]

- ☐ A One to the side of the track
- ☐ B One straight ahead
- ☐ C One directly behind
- ☐ D Any landmark helps equally

Answer: A. A range to a landmark to the side measures across-track position. Choosing routes to keep the ellipse small is active perception.

The task: the shape of the uncertainty

Drive at least 60 cm, growing an along-track and an across-track uncertainty as you go. Plot both, and print `along:` and `across:` at the end.

```
from bugbot import *
import math
connect()

DT = 0.1
along = across = 0.0
head_sigma = 0.5
```

The hint students can ask for: Grow two numbers as the robot drives: the along-track uncertainty by the scale error times the distance travelled each tick, and the across-track by the distance times the heading uncertainty in radians. Plot both and watch the ellipse stretch.

A solution

```
from bugbot import *
import math
connect()

DT = 0.1
SCALE_ERR = 0.04          # the flow sensor's scale, a few percent
HEAD_ERR = 0.03           # radians of heading uncertainty, growing slowly

along = across = 0.0
head_sigma = 0.5
forward(70)
for i in range(60):
    stepped = abs(flow()[1]) * DT
    head_sigma += HEAD_ERR * DT * 10
    along += SCALE_ERR * stepped
    across += stepped * math.radians(head_sigma)
    plot("along", along)
    plot("across", across)
    wait(DT)
stop()
print("along:", round(along, 2))
print("across:", round(across, 2))
```

Any program that meets the task's checks is marked correct in the simulator; this is one way, not the only way.